Atoms

1. The transition of electron that gives rise to the formation of the second spectral line of the Balmer series in the spectrum of hydrogen atom corresponds to: (2024)

(A) $n_f = 2$ and $n_i = 3$

(B) $n_f = 3$ and $n_i = 4$

(C) $n_f = 2$ and $n_i = 4$

(D) $n_f = 2$ and $n_i = \infty$

Ans. (C) $n_f = 2$ and $n_i = 4$

2. Write the drawbacks of Rutherford's atomic model. How did Bohr remove them? Show that different orbits in Bohr's atom are not equally spaced. (2024)

Ans.

- Drawbacks of Rutherford's atomic model
- Bohr's explanation
- Showing different orbits are not equally spaced

Drawbacks:

(i) According to classical electromagnetic theory, an accelerating charged particle emits radiation in the form of electromagnetic waves. The energy of an accelerating electron should therefore, continuously decrease. The electron would spiral inward and eventually fall into the nucleus. Thus, such an atom cannot be stable.

(ii) As the electrons spiral inwards, their angular velocities and hence their frequencies would change continuously. Thus, they would emit a continuous spectrum, in contradiction to the line spectrum actually observed.

Bohr postulated stable orbits in which electrons do not radiate energy
Alternatively:

Bohr's postulates (Any ONE of the three)

(i) An electron in an atom could revolve in certain stable orbits without the emission of radiant energy.

(ii) The electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $h/2\pi$

(iii) An electron might make a transition from one of its specified nonradiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states.

The radius of the n^{th} orbit is found as

$$r_n = \left(\frac{n^2}{m}\right)\left(\frac{h}{2\pi}\right)^2 \frac{4\pi\varepsilon_0}{e^2}$$

$r_n \propto n^2$

Alternatively:

Difference in radius of consecutive orbits is

$r_n+1 - r_n = k [(n+1)^2 - n^2)]$

$= k (2n + 1)$ which depends on n, and is not a constant

Previous Years' CBSE Board Questions

12.2 Alpha-Particle Scattering and Rutherford's Nuclear Model of Atom

MCQ

1. Which of the following statements is not correct according to Rutherford model?
   (a) Most of the space inside an atom is empty.
   (b) The electrons revolve around the nucleus under the influence of coulomb force acting on them.
   (c) Most part of the mass of the atom and its positive charge are concentrated at its centre.
   (d) The stability of atom was established by the model. *(2020)* R

SA I (2 marks)

2. (a) In Geiger-Marsden experiment, the distance of closest approach is considerably larger than the sum of the radii of the gold nucleus and the α-particle. Explain.
   (b) The total energy of hydrogen atom in a state is –3.4 eV. What are the kinetic and potential energies of the electron in this state? *(2022C)*

3. What result do you expect if α-particle scattering experiment is repeated using a thin sheet hydrogen in place of a gold foil? Explain. (Hydrogen is a solid at temperature below 14 K) *(Term II 2021-22)* U

4. In the Geiger-Marsden experiment, an α-particle of 5.12 MeV energy approaches a gold target (Z = 79), comes momentarily to rest and then reverses its direction. Find the distance of closest approach of α-particle to the target nucleus. *(2020)*

6. Define the distance of closest approach. An α-particle of kinetic energy 'K' is bombarded on a thin gold foil. The distance of the closest approach is 'r'. What will be the distance of closest approach for an α-particle of double the kinetic energy? *(Delhi 2017)* An

7. Write two important limitations of Rutherford nuclear model of the atom. *(Delhi 2017)* R

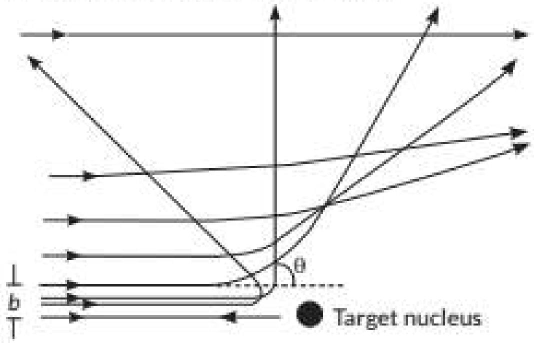
8. Using Rutherford's model of the atom, derive the expression for the total energy of the electron in hydrogen atom. What is the significance of total negative energy possessed by the electron? *(AI 2014)* Ev

SA II (3 marks)

9. (a) The energy of hydrogen atom in an orbit is –1.51 eV. What are kinetic and potential energies of the electron in this orbit?
   (b) The electron in a hydrogen atom is typically found at a distance of about 5.3×10^{-11} m from the nucleus which has a diameter of about 1.0×10^{-15} m. Assuming the hydrogen atom to be a sphere of radius 5.3×10^{-11} m, what fraction of its volume is occupied by the nucleus. *(Term II 2021-22)* Ev

10. (a) In the Geiger-Marsden experiment, find the distance of closest approach to the gold nucleus (mass number = 79) of a 7.7 MeV α-particle before it comes momentarily to rest and reverses its direction. Why is it different from actual radius of gold nucleus? *(2021)*
    (b) Plot a graph between number of scattered α-particles detected in gold foil experiment and angle of scattering. What is the main assumption in plotting this graph? *(2021C)*

5. In Geiger-Marsden scattering experiment, the trajectory of α-particles in Coulomb's field of a heavy nucleus is shown in the figure.



(a) What do b and θ represent in the figure?

(b) What will be the value of b for (i) $\theta = 0°$, (ii) $\theta = 180°$? *(2020)*

LA **(5 marks)**

14. In Rutherford scattering experiment, draw the trajectory traced by α-particles in the coulomb field of target nucleus and explain how this led to estimate the size of the nucleus. *(3/5, AI 2015C)* An

12.4 Bohr Model of the Hydrogen Atom

MCQ

15. The radius of the n^{th} orbit in Bohr model of hydrogen atom is proportional to

(a) n^2 (b) $\frac{1}{n^2}$ (c) n (d) $\frac{1}{n}$

(2023) U

SA I **(2 marks)**

16. Using Bohr's atomic model, derive the expression for the radius of n^{th} orbit of the revolving electron in a hydrogen atom. *(2020)* An

OR

Show that the radius of the orbit in hydrogen atom varies as n^2, where n is the principal quantum number of the atom. *(Delhi 2015)*

OR

11. (a) In Geiger-Marsden experiment, calculate the distance of closest approach for an alpha particle with energy 2.56×10^{-12} J. Consider that the particle approaches gold nucleus ($Z = 79$) in head-on position.

(b) If the above experiment is repeated with a proton of the same energy, then what will be the value of the distance of closest approach? *(Term II 2021-22)*

12. Draw a graph showing the variation of number of particles scattered (N) with the scattering angle θ in Geiger-Marsden experiment. Why only a small fraction of the particles are scattered at $\theta > 90°$? Mention two limitations of Rutherford nuclear model of an atom. *(Term II 2021-22)*

13. Explain briefly how Rutherford scattering of α-particle by a target nucleus can provide information of the size of the nucleus. *(2/3, Delhi 2019)* U

(i) the maximum, and

(ii) the minimum wavelength in Balmer series of hydrogen spectrum. *(2022C)*

22. Using Bohr's postulates, derive the expression for the orbital period of the electron moving in the n^{th} orbit of hydrogen atom. *(2/3, AI 2019)* U

23. A hydrogen atom initially in the ground state absorbs a photon which excites it to the $n = 4$ level. Estimate the frequency of the photon. *(1/3, 2018)*

24. (a) The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. Calculate its radius in $n = 3$ orbit.

(b) The total energy of an electron in the first excited state of the hydrogen atom is 3.4 eV. Find out its (i) kinetic energy and (ii) potential energy in this state. *(Delhi 2014C)* Ev

LA **(5 marks)**

25. (a) Write two important limitations of Rutherford model which could not explain the observed features of atomic spectra. How were these explained in Bohr's model of hydrogen atom?

(b) Using Bohr's postulates, obtain the expression for the radius of the n^{th} orbit in hydrogen atom. *(4/5, Delhi 2015C)* An

Using Bohr's postulates of the atomic model, derive the expression for the radius of n^{th} electron orbit. Hence obtain the expression for Bohr's radius. *(AI 2014)*

17. Write shortcomings of Rutherford atomic model. Explain how these were overcome by the postulates of Bohr's atomic model. *(2020)* U

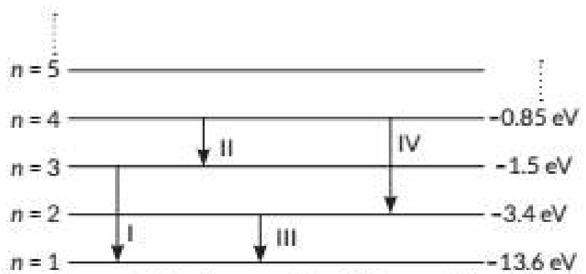
18. State Bohr's quantization condition of angular momentum. Calculate the shortest wavelength of the Brackett series and state to which part of the electromagnetic spectrum does it belong. *(Delhi 2019)*

19. Find out the wavelength of the electron orbiting in the ground state of hydrogen atom. *(Delhi 2017)* U

20. State Bohr postulate of hydrogen atom that gives the relationship for the frequency of emitted photon in a transition. *(Foreign 2016)*

SA II (3 marks)

21. (a) How did de Broglie hypothesis provide an explanation for Bohr's second postulate for quantisation of orbital angular momentum of the orbiting electron in hydrogen atom? Discuss.
(b) Identify the transition of electron in Bohr model which gives rise to



(a) I (b) II (c) III (d) IV *(2020)*

SA I (2 marks)

30. Name the spectral series for a hydrogen atom which lies in the visible region. Find the ratio of the maximum to the minimum wavelength of this series. *(Term II 2021-22)*

31. Calculate the orbital period of the electron in the first excited state of hydrogen atom. *(Delhi 2019)* U

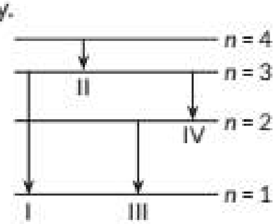
26. Using Bohr's postulates, derive the expression for the total energy of the electron in the stationary states of the hydrogen atom. *(3/5, Foreign 2014)*

12.5 The Line Spectra of the Hydrogen Atom

MCQ

27. The potential energy of an electron in the second excited state in hydrogen atom is
(a) −3.4 eV (b) −3.02 eV
(c) −1.51 eV (d) −6.8 eV *(2023)* Ap

28. The diagram shows four energy level of an electron in Bohr model of hydrogen atom. Identify the transition in which the emitted photon will have the highest energy.



(a) I (b) II
(c) III (d) IV *(2023)*

29. The figure shows the energy level diagram of hydrogen atom with few transitions. Which transition shows the emission of photon with maximum energy?

SA II (3 marks)

38. An electron in a hydrogen atom makes transitions from orbits of higher energies to orbits of lower energies.
(i) When will such transitions result in (a) Lyman (b) Balmer series?
(ii) Find the ratio of the longest wavelength in Lyman series to the shortest wavelength in Balmer series. *(Term II 2021-22)*

39. Write Rydberg's formula for wavelengths of the spectral lines of hydrogen spectrum. Mention to which series in the emission spectrum of hydrogen, H_α line belongs. *(1/3, AI 2019)* Ev

40. Using Rydberg formula, calculate the longest wavelength belonging to Lyman and Balmer series of hydrogen spectrum. In which region these transitions lie? *(3/5, Foreign 2015)*

OR

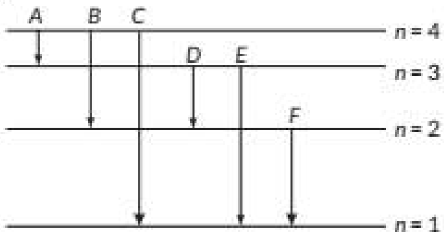
32. A 12.75 eV electron beam is used to excite a gaseous hydrogen atom at room temperature. Determine the wavelengths and the corresponding series of the lines emitted. *(AI 2017)* U

33. The ground state energy of hydrogen atom is –13.6 eV. If an electron makes a transition from an energy level – 1.51 eV to – 3.4 eV, calculate the wavelength of the spectral line emitted and name the series of hydrogen spectrum to which it belongs. *(AI 2017)* Ap

34. Define ionization energy. How would the ionization energy change when electron in hydrogen atom is replaced by a particle of mass 200 times that of the electron but having the same charge? *(AI 2016)*

35. An electron jumps from fourth to first orbit in an atom. How many maximum number of spectral lines can be emitted by the atom? To which series these lines correspond? *(Foreign 2016)*

36. The figure shows energy level diagram of hydrogen atom.



(a) Find out the transition which results in the emission of a photon of wavelength 496 nm.

(b) Which transition corresponds to the emission of radiation of maximum wavelength? Justify your answer. *(AI 2015C)* Ap

37. Calculate the shortest wavelength in the Balmer series of hydrogen atom. In which region (infrared, visible, ultraviolet) of hydrogen spectrum does this wavelength lie? *(AI 2015)*

Using Rydberg formula, calculate the wavelengths of the spectral lines of the first member of the Lyman series and of the Balmer series. *(2/5, Foreign 2014)*

41. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. Upto which energy level the hydrogen atoms would be excited?

Calculate the wavelengths of the first member of Lyman and first member of Balmer series. *(Delhi 2014)* An

42. The value of ground state energy of hydrogen atom is –13.6 eV.

(i) Find the energy required to move an electron from the ground state to the first excited state of the atom.

(ii) Determine (a) the kinetic energy and (b) orbital radius in the first excited state of the atom.

(Given the value of Bohr radius = 0.53 Å).

(AI 2014C) Cr

12.6 de-Broglie's Explanation of Bohr's Second Postulate of Quantisation

VSA (1 mark)

43. According to Bohr's atomic model, the circumference of the electron orbit is always an ____ multiple of de-Broglie wavelength. *(2020)*

SA I (2 marks)

44. Obtain the expression for the ratio of the de-Broglie wavelengths associated with the electron orbiting in the second and third excited states of hydrogen atom. *(Delhi 2019)*

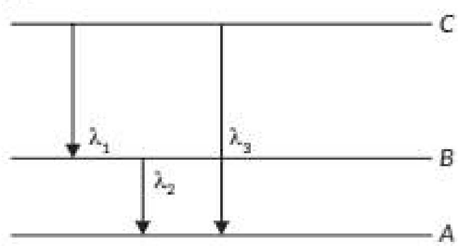
45. Calculate the de-Broglie wavelength of the electron orbiting in the $n = 2$ state of hydrogen atom. *(AI 2016)* Ev

46. Use de-Broglie's hypothesis to write the relation for the n^{th} radius of Bohr orbit in terms of Bohr's quantization condition of orbital angular momentum. *(Foreign 2016)*

SA II (3 marks)

47. Calculate the de-Broglie wavelength associated with the electron in the 2^{nd} excited state of hydrogen atom. The ground state energy of the hydrogen atom is 13.6 eV. *(2020)*

(ii) Find the relation between the three wavelengths λ_1, λ_2 and λ_3 from the energy level diagram shown in the figure.



48. State Bohr's postulate to define stable orbits in hydrogen atom. How does de-Broglie's hypothesis explain the stability of these orbits? *(2/3, 2018)* An

49. (i) State Bohr's quantization condition for defining stationary orbits. How does de-Broglie hypothesis explain the stationary orbits ? *(Delhi 2016)* Cr

50. The kinetic energy of the electron orbiting in the first excited state of hydrogen atom is 3.4 eV. Determine the de-Broglie wavelength associated with it. *(Foreign 2015)*

CBSE Sample Questions

12.4 Bohr Model of the Hydrogen Atom

MCQ

1. The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. The radius of the n =3 orbit is

(a) 1.01×10^{-10} m (b) 1.59×10^{-10} m
(c) 2.12×10^{-10} m (d) 4.77×10^{-10} m

(2022-23)

VSA (1 mark)

2. Consider two different hydrogen atoms. The electron in each atom is in an excited state. Is it possible for the electrons to have different energies but same orbital angular momentum according to the Bohr model? Justify your answer. *(Term II 2021-22)* U

3. What is the value of angular momentum of electron in the second orbit of Bohr's model of hydrogen atom? *(2020-21)* R

SA I (2 marks)

4. Derive an expression for the radius of n^{th} Bohr's orbit in Hydrogen atom. *(2019-20)*

5. Energy of electron in first excited state in Hydrogen atom is –3.4 eV. Find K.E. and P.E. of electron in the ground state. *(2019-20)* Ap

SA II (3 marks)

6. The ground state energy of hydrogen atom is –13.6 eV. The photon emitted during the transition of electron from n = 3 to n = 1 state, is incident on a photosensitive material of unknown work function. The photoelectrons are emitted from the material with the maximum kinetic energy of 9 eV. Calculate the threshold wavelength of the material used. *(2022-23)*

12.5 The Line Spectra of the Hydrogen Atom

MCQ

7. A photon beam of energy 12.1 eV is incident on a hydrogen atom. The orbit to which electron of H-atom be excited is

(a) 2^{nd} (b) 3^{rd}
(c) 4^{th} (d) 5^{th}

(2019-20) R

SA I (2 marks)

8. The short wavelength limit for the Lyman series of the hydrogen spectrum is 913.4 Å. Calculate the short wavelength limit for the Balmer series of the hydrogen spectrum. *(2022-23)*

SA II (3 marks)

9. Derive an expression for the frequency of radiation emitted when a hydrogen atom de-excites from level n to level (n – 1). Also show that for large values of n, this frequency equals to classical frequency of revolution of an electron. *(Term II 2021-22, 2020-21)*

Detailed SOLUTIONS

Previous Years' CBSE Board Questions

1. (d)

2. (a) In the Geiger-Marsden experiment, alpha particles were made to fall on gold foil. In this experiment, Rutherford made some assumption, in which one of assumption is–

The nucleus and alpha particle both are point charges having no dimensions.

While the distance of closest approach is given by

$$r_0 = \frac{2Ze^2}{4\pi\varepsilon_0 k}$$

which will be considerably larger than the sum of radii of gold nucleus and α-particle being these point charges.

(b) Energy of electron in $n = 2$ is −3.4 eV

K.E. = −T.E. = +13.6 eV

$$E_n = \frac{x}{n^2} \Rightarrow -3.4\,\text{eV} = \frac{x}{2^2}$$

$\Rightarrow$ $x = -13.6$ eV

Energy in ground state, $x = -13.6$ eV

P.E. = 2T.E. = −2 × 13.6 = −27.2 eV

3. In the α-particle scattering experiment, if a thin sheet of solid hydrogen is used in place of a gold foil, then the scattering angle would not be large enough because the mass of hydrogen (1.67×10^{-27}) is less than the mass of incident α-particle (6.64×10^{-27}). Thus, the mass of scattering particle is more than the target nucleus. As a result, α-particles would not bounce back if solid hydrogen is used in the α-particle scattering.

4. Here, $K = 5.12$ MeV $= 5.12 \times 1.6 \times 10^{-13}$ J

Z (Au) = 79

At the distance of closest approach of an α-particle

$$r_0 = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{K} = 444.375 \times 10^{-16}\,\text{m}$$

$$\simeq 44.44\,\text{fm}$$

5. (a) In the Geiger-Marsden scattering experiment, b represents the impact parameter and θ represents the scattering angle.

$$r = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{K} \text{ or } r \propto \frac{1}{K}$$

If kinetic energy will be doubled, then the distance of closest approach will become half.

Concept Applied

- By conservation of energy, initial kinetic energy of α-particle would be converted into electrostatics potential energy of α-particle and nucleus at distance r.

7. The two important limitations of Rutherford nuclear model of the atom are :

(i) This model cannot explain about the stability of matter.

(ii) It cannot explain the characteristic line spectra of atoms of different elements.

8. An electron revolving in an orbit of H-atom, has both kinetic energy and electrostatic potential energy. Kinetic energy of the electron revolving in a circular orbit of radius r is $E_K = \frac{1}{2}mv^2$

Since, $\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$

$$\therefore \quad E_K = \frac{1}{2} \times \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\text{or } E_K = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r} \quad \text{... (i)}$$

Electrostatic potential energy of electron of charge $-e$ revolving around the nucleus of charge $+e$ in an orbit of radius r is

$$E_P = \frac{1}{4\pi\varepsilon_0}\frac{(+e)\times(-e)}{r}$$

$$\text{or } E_P = \frac{-1}{4\pi\varepsilon_0}\frac{e^2}{r} \quad \text{... (ii)}$$

So, total energy of electron in orbit of radius r is

$$E = E_K + E_P \text{ or } E = \left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r}\right) - \left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}\right)$$

$$\text{or } E = \frac{-1}{4\pi\varepsilon_0}\frac{e^2}{2r}$$

9.

(a) From Bohr's postulate, we have

$\frac{Ke^2}{r^2} = \frac{mv^2}{r}$ [1st postulate] [Electrostatic force applies the required centripetal force]

$K = \frac{1}{4\pi\varepsilon_0} = 9\times10^9$

e = charge of electron
r = radius of orbit
m = mass of electrons
v = velocity of electrons in the orbit.

$\Rightarrow mv^2 = \frac{Ke^2}{r}$

Now Kinetic energy of electron in orbit (KE) $= \frac{1}{2}mv^2 = \frac{Ke^2}{2r} = KE$

Potential energy of electron in the orbit (PE) $= \frac{K(e)(-e)}{r} = \frac{-Ke^2}{r} = P.E$

Total energy of electron (TE) = PE + KE

$\Rightarrow TE = -\frac{Ke^2}{r} + \frac{Ke^2}{2r}$

$TE = -\frac{Ke^2}{2r}$

Given that energy of electron in Hydrogen atom is $= -1.51\ eV = -\frac{Ke^2}{2r}$

$\Rightarrow \frac{Ke^2}{r} = 3.02\ eV$

$\Rightarrow PE = -\frac{Ke^2}{r} = -3.02\ eV$, $KE = \frac{Ke^2}{2r} = +1.51\ eV$

Ans. Thus, KE = +1.51 eV
PE = −3.02 eV

(b) Given radius of atom (r_a) = radius of electron orbit $= 5.3\times10^{-11}$ m

radius of nucleus (r_n) = $\frac{\text{diameter}}{2} = \frac{1.0\times10^{-15}\ m}{2} = 5\times10^{-16}$ m

Volume of atom $= \frac{4}{3}\pi r_a^3$

Volume of nucleus $= \frac{4}{3}\pi r_n^3$

⇒ Fraction of its volume occupied by the nucleus $= \frac{(\text{Volume})_{\text{nucleus}}}{(\text{Volume})_{\text{atom}}} = \frac{\frac{4}{3}\pi r_n^3}{\frac{4}{3}\pi r_a^3}$

$= \left(\frac{r_n}{r_a}\right)^3$

$= \left(\frac{5}{5.3}\times10^{-5}\right)^3$

$= \frac{125}{148.877}\times10^{-15}$

$= 8.395\times10^{-16}$

Ans. Thus, fraction of volume of hydrogen atom occupied by its nucleus $= 8.395\times10^{-16}$

system consisting of an α-particle and a gold nucleus is conserved. The system's initial mechanical energy is E_i, before the particle and nucleus interact, and it is equal to its final mechanical energy E_f when the α-particle momentarily stops. The initial energy E_i is just the kinetic energy K of the incoming α-particle. The final energy E_f is just the electric potential energy U of the system.

Let d be the centre-to-centre distance between the α-particle and the gold nucleus when the α-particle is at its stopping point. Then we can write the conservation of energy $E_i = E_f$ as

$$K = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(Ze)}{d} = \frac{2Ze^2}{4\pi\varepsilon_0 d}$$

Thus the distance of closest approach d is given by

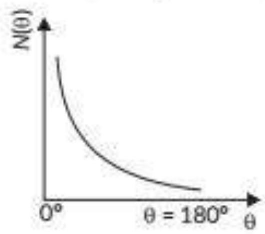
$$d = \frac{2Ze^2}{4\pi\varepsilon_0 K}$$

The maximum kinetic energy found in α-particles of natural origin is 7.7 MeV or 1.2×10^{-12} J. Since $1/4\pi\varepsilon_0 = 9.0\times10^9\ N\ m^2/C^2$. Therefore with $e = 1.6\times10^{-19}$ C, we have,

$$d = \frac{(2)(9.0\times10^9 Nm^2/C^2)(1.6\times10^{-19}C)^2 Z}{1.2\times10^{-12} J} = 3.84\times10^{-16}\ Z\,m$$

The atomic number of gold is $Z = 79$, so that d (Au) $= 3.0\times10^{-14}$ m = 30 fm. (1 fm *i.e.*, fermi $= 10^{-15}$ m.)

(b)



This graph shows deflection of number of particles with angle of deflection θ.

The main assumption is that the target atom has a small, dense, positively charged nucleus.

11. (a) Let the minimum distance of approach be r_0. At this distance, the whole of the kinetic energy of the alpha-particle will be converted into the electrical potential energy.

The positive charge on the gold nucleus = $Ze = 79e$ and the positive charge on the α-particle = $2e$

At $r = r_0$, KE = PE

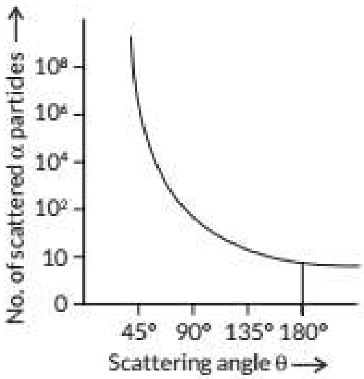
$$2.56\times10^{-12} = \frac{1}{4\pi\varepsilon_0}\frac{(79e)(2e)}{r_0}$$

$$\therefore r_0 = \frac{(9\times10^9)(79)(2)(1.6\times10^{-19})^2}{2.56\times10^{-12}} = 14.2\times10^{-15}\ m$$

[Topper's Answer, 2022]

10. (a) The key idea here is that throughout the scattering process, the total mechanical energy of the

12. A very small fraction of α-particles are scattered at θ > 90° because the size of nucleus is very small nearly 1/8000 times the size of atom. So, a few α-particles experience a strong repulsive force and turn back.

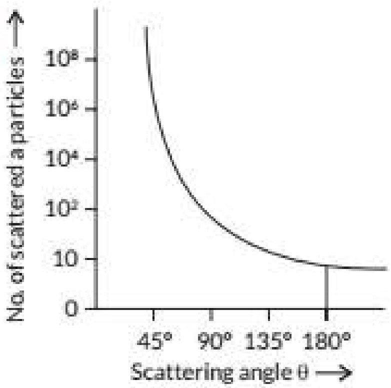


The two important limitations of Rutherford nuclear model of the atom are :

(i) This model cannot explain about the stability of atom.

(ii) It cannot explain the characteristic line spectra of atoms of different elements.

13.



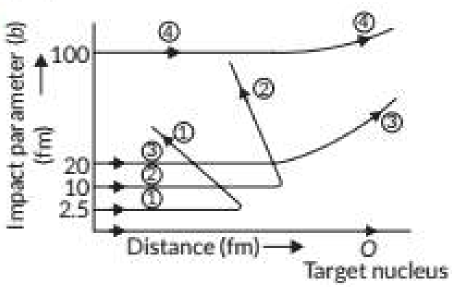
A very small fraction of α-particles are scattered at θ > 90° because the size of nucleus is very small nearly 1/8000 times the size of atom. So, a few α-particles experience a strong repulsive force and turn back.

Conclusions :

(i) Entire positive charge and most of the mass of the atom is concentrated in the nucleus with the electrons some distance away.

(ii) Size of the nucleus is about 10^{-15} m to 10^{-14} m, while size of the atom is 10^{-10} m, so the electrons are at distance 10^4 m to 10^5 m from the nucleus, and being large empty space in the atom, most α particles go through the empty space.

14.



(b) If proton is used instead of alpha particle, r_0 will become half.

$\therefore \quad r_0' = \frac{r_0}{2} = 7.1 \times 10^{-15}$ m

where, E = K.E of α-particle

θ = scattering angle

Z = atomic number of atom

The size of the nucleus is smaller than the impact parameter.

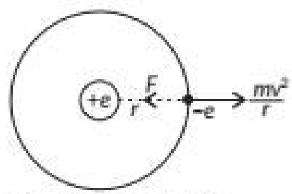
Key Points

- The real size of nucleus is smaller than impact parameter.

15. (a): radius, $r = n^2 r_0$

16. Radius of n^{th} orbit of hydrogen atom : In H-atom, an electron having charge $-e$ revolves around the nucleus of charge +e in a circular orbit of radius r, such that necessary centripetal force is provided by the electrostatic force of attraction between the electron and nucleus.

$$i.e., \quad \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e.e}{r^2} \quad \text{or} \quad mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \quad \text{...(i)}$$



From Bohr's quantization condition

$$mvr = \frac{nh}{2\pi} \quad \text{or} \quad v = \frac{nh}{2\pi mr} \quad \text{...(ii)}$$

Using equation (ii) in (i), we get

$$m\cdot\left(\frac{nh}{2\pi mr}\right)^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \quad \text{or} \quad \frac{m\cdot n^2h^2}{4\pi^2m^2r^2} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\text{or} \quad r = \frac{n^2h^2\varepsilon_0}{\pi me^2} \quad \text{...(iii)}$$

where n = 1, 2, 3, ... is principal quantum number.

Equation (iii), gives the radius of n^{th} orbit of H-atom. So the radii of the orbits increase proportionally with n^2 i.e., [$r \propto n^2$]. Radius of first orbit of H-atom is called Bohr radius a_0 and is given by

$$a_0 = \frac{h^2\varepsilon_0}{\pi me^2} \text{ for } n = 1 \text{ or } a_0 = 0.529 \text{ Å}$$

So, radius of n^{th} orbit of H-atom then becomes

$r = n^2 \times 0.529$ Å

17. Limitation of Rutherford's model :

Rutherford's atomic model is inconsistent with classical physics, that is why, Rutherford's model is not able to explain the spectrum of even most simplest H-spectrum.

Bohr's postulates to resolve observed features of atomic spectrum :

The size of the nucleus can be obtained by finding impact parameter b using trajectories of α-particle. The impact parameter is the perpendicular distance of the initial velocity vector of α-particle from the central line of nucleus, when it is far away from the atom. Rutherford calculated impact parameter as

$$b=\frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2\cot(\theta/2)}{E}$$

Bohr's quantization condition: Of all the possible circular orbits allowed by the classical theory, the electrons are permitted to circulate only in those orbits in which the angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$, h being Planck's constant. Therefore, for any permitted orbit,

$$L=mvr=\frac{nh}{2\pi},\ n=1,2,3.....,$$

where n is called the principal quantum number, and this equation is called Bohr's quantisation condition.

18. Bohr's quantization condition : The electron can revolve around the nucleus only in those circular orbits in which angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$ *i.e.*,

$$mvr=\frac{nh}{2\pi}, n=1,2,3,....$$

The shortest wavelength of Brackett series is given as

$$\frac{1}{\lambda}=1.097\times10^7\left[\frac{1}{4^2}-\frac{1}{\infty^2}\right]=\frac{1.097\times10^7}{16}$$

$\Rightarrow\ \lambda = 1.4585\times10^{-6}$ m

This wavelength lies in the infrared region of electromagnetic spectrum.

19. The wavelength of the electron orbiting in the ground state of hydrogen atom is

$$\lambda=\frac{hc}{E}=\frac{6.62\times10^{-34}\times3\times10^8}{-13.6\,\text{eV}}$$

($\because$ Ground state energy of hydrogen atom = –13.6 eV)

$$=\frac{6.62\times10^{-34}\times3\times10^8}{-13.6\times1.6\times10^{-19}}$$

$$=\frac{-1.986\times10^{-25}}{21.76\times10^{-19}}=9.126\times10^{-8}\text{ m}=912.6\text{ Å}$$

20. Frequency condition of Bohr model : An atom can emit or absorb radiation in the form of discrete energy photons only when an electron jumps from a higher to a lower orbit or from a lower to a higher orbit, respectively.

$h\upsilon = E_i - E_f$

where υ is frequency of radiation emitted, E_i and E_f are the energies associated with stationary orbits of principal quantum number n_i and n_f respectively (where $n_i > n_f$).

21. Bohr second postulate : It states that the angular momentum of the electron orbiting around the nucleus is quantised (that is, $L_n = nh/2\pi$; n = 1, 2, 3 ...). According to de-Broglie the electron in its circular orbit, as proposed by Bohr, must be seen as a particle wave.

(b) Series limit line (shortest wavelength) of Balmer series given by

$$\frac{1}{\lambda}=R\left[\frac{1}{2^2}-\frac{1}{\infty^2}\right]=\frac{R}{4}\text{ or }\lambda=\frac{4}{R}$$

The first line (longest wavelength) of the Balmer series is given by

$$\frac{1}{\lambda}=R\left[\frac{1}{2^2}-\frac{1}{3^2}\right]=\frac{5R}{36}\text{ or }\lambda=\frac{36}{5R}$$

22. When the electron revolves around the nucleus in a certain specified circular orbits without the emission of radiant energy.

Then, centripetal force = electrostatic force

$$\frac{mv_n^2}{r_n}=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2} \quad ...(i)$$

Now, from Bohr's postulate of angular momentum

$$mv_nr_n=\frac{nh}{2\pi} \quad ...(ii)$$

From (i) and (ii) we get,

$$r_n=\frac{n^2h^2\varepsilon_0}{\pi me^2}\text{ and }v_n=\frac{e^2}{2\varepsilon_0 nh}$$

$\therefore$ Time period of revolution of electron in n^{th} orbit,

$$T_n=\frac{2\pi r_n}{v_n}=\frac{4\varepsilon_0^2n^3h^3}{me^4}$$

Concept Applied

- Here, centripetal force is provided by electrostatic force.

23. Energy of hydrogen atom in n^{th} state

$$E_n=-\frac{13.6\,\text{eV}}{n^2}$$

According to question, $h\upsilon = E_4 - E_1$

$$h\upsilon=-13.6\left(\frac{1}{16}-1\right)\text{eV}=13.6\times\frac{15}{16}\text{eV}$$

$$\upsilon=13.6\times\frac{15}{16}\times\frac{1.6\times10^{-19}}{6.63\times10^{-34}}=3\times10^{15}\text{Hz}$$

When a string is plucked, vast number of wavelengths are excited. However only those wavelength survived which have nodes at the ends and form the standing wave in the string.

According to de-Broglie hypothesis, a stationary orbit is the one that contains an integral number of de-Broglie waves associated with the revolving electron.

Total distance covered by electron = Circumference of the orbit

i.e., $2\pi r_n = n\lambda$, n = 1,2,3.... ...(i)

Now, according to De-Broglie wavelength, $\lambda = \frac{h}{mv}$

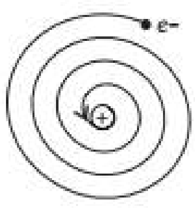
Now put it in eqn. (i), we have, $2\pi r_n = \frac{nh}{mv_n}$

$mv_n r_n = \frac{nh}{2\pi}$, which is the required quantum condition proposed by Bohr for the angular momentum of the electron.

(ii) Potential energy,

$E_p = 2E = -3.4 \times 2 = -6.8$ eV

25. (a) Limitation of Rutherford's model :



Rutherford's atomic model is inconsistent with classical physics. According to electromagnetic theory, an electron is a charged particle moving in the circular orbit around the nucleus and is accelerated, so it should emit radiation continuously and thereby loose energy. Due to this, radius of the electron would decrease continuously and also the atom should then produce continuous spectrum, and ultimately electron will fall into the nucleus and atom will collapse in 10^{-8} s. But the atom is fairly stable and it emits line spectrum.

(ii) Rutherford's model is not able to explain the spectrum of even most simplest H-spectrum.

Bohr's postulates to resolve observed features of atomic spectrum :

(i) Quantum condition: Of all the possible circular orbits allowed by the classical theory, the electrons are permitted to circulate only in those orbits in which the angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$,

h being Planck's constant. Therefore, for any permitted orbit,

$$L = mvr = \frac{nh}{2\pi},\ n = 1, 2, 3.....,$$

where n is called the principal quantum number, and this equation is called Bohr's quantisation condition.

(ii) Stationary orbits: While resolving in the permissible orbits, an electron does not radiate energy. These non-radiating orbits are called stationary orbits.

Commonly Made Mistake

- Students consider the e^- in an H-atom will absorb any amount of energy, some energy will be used in jumping the e^- to higher energy rest is used as kinetic energy. But that is not true, Electron will only absorb the energy to jump equivalent to allowed energy levels.

24. (a) Radius of n^{th} orbit, $r_n \propto n^2$

$\Rightarrow \frac{r_3}{r_1} = \frac{3^2}{1^2} = 9$ or $r_3 = 9r_1 = 9 \times 5.3 \times 10^{-11}$

$= 47.7 \times 10^{-11}$ m $= 4.77 \times 10^{-10}$ m

(b) (i) Kinetic energy,

$E_k = -E = -(-3.4) \times 1 = 3.4$ eV

$$mvr = \frac{nh}{2\pi} \text{ or } v = \frac{nh}{2\pi mr} \quad ...(ii)$$

Using equation (ii) in (i), we get

$$m \cdot \left(\frac{nh}{2\pi mr}\right)^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \text{ or } \frac{m \cdot n^2h^2}{4\pi^2 m^2 r^2} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\text{or} \quad r = \frac{n^2h^2\varepsilon_0}{\pi me^2} \quad ...(iii)$$

where n = 1, 2, 3, ... is principal quantum number.

Equation (iii), gives the radius of n^{th} orbit of H-atom. So the radii of the orbits increase proportionally with n^2 *i.e.*, $[r \propto n^2]$. Radius of first orbit of H-atom is called Bohr radius a_0 and is given by

$$a_0 = \frac{h^2\varepsilon_0}{\pi me^2} \text{ for } n = 1 \text{ or } a_0 = 0.529 \text{ Å}$$

So, radius of n^{th} orbit of H-atom then becomes

$r = n^2 \times 0.529$ Å

26. (i) According to Bohr's postulates, in a hydrogen atom, as single electron revolves around a nucleus of charge $+e$. For an electron moving with a uniform speed in a circular orbit of a given radius, the centripetal force is provided by coulomb force of attraction between the electron and the nucleus. The gravitational attraction may be neglected as the mass of electron and proton is very small.

$$\text{So, } \frac{mv^2}{r} = \frac{ke^2}{r^2} \quad \left(\text{Where, } k = \frac{1}{4\pi\varepsilon_0}\right)$$

$$\text{or} \quad mv^2 = \frac{ke^2}{r} \quad ...(i)$$

Where, m = mass of electron

r = radius of electronic orbit

v = velocity of electron

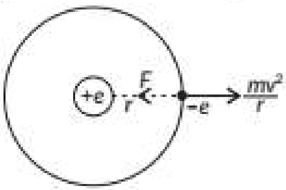
(iii) Frequency condition: An atom can emit or absorb radiation in the form of discrete energy photons only when an electron jumps from a higher to a lower orbit or from a lower to a higher orbit, respectively.

$h\upsilon = E_i - E_f$

where υ is frequency of radiation emitted, E_i and E_f are the energies associated with stationary orbits of principal quantum number n_i and n_f respectively (where $n_i > n_f$).

(b) Radius of n^{th} orbit of hydrogen atom : In H-atom, an electron having charge $-e$ revolves around the nucleus of charge $+e$ in a circular orbit of radius r, such that necessary centripetal force is provided by the electrostatic force of attraction between the electron and nucleus.

i.e., $\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e.e}{r^2}$ or $mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$ ...(i)



From Bohr's quantization condition

Again, by Bohr's second postulates

$mvr = \frac{nh}{2\pi}$

Where, $n = 1, 2, 3......$ or $v = \frac{nh}{2\pi mr}$

Putting the value of v in eq. (i)

$m\left(\frac{nh}{2\pi mr}\right)^2 = \frac{ke^2}{r} \Rightarrow r = \frac{n^2h^2}{4\pi^2kme^2}$ ...(ii)

Kinetic energy of electron,

$E_k = \frac{1}{2}mv^2 = \frac{ke^2}{2r}$ $\left(\because \frac{mv^2}{r} = \frac{ke^2}{r^2}\right)$

Using eq. (ii) we get

$E_k = \frac{ke^2}{2}\frac{4\pi^2kme^2}{n^2h^2} = \frac{2\pi^2k^2me^4}{n^2h^2}$

Potential energy of electron,

$E_p = -\frac{k(e)\times(e)}{r} = -\frac{ke^2}{r}$

Using eq. (ii), we get

$E_p = -ke^2 \times \frac{4\pi^2kme^2}{n^2h^2} = -\frac{4\pi^2kme^4}{n^2h^2}$

Hence, total energy of the electron in the n^{th} orbit

$E = E_p + E_k$

$= -\frac{4\pi^2k^2me^4}{n^2h^2} + \frac{2\pi^2k^2me^4}{n^2h^2} = -\frac{2\pi^2k^2me^4}{n^2h^2} = -\frac{13.6}{n^2}$ eV

When the electron in a hydrogen atom jumps from higher energy level to the lower energy level, the difference of energies of the two energy levels is emitted as a radiation of particular wavelength. It is called a spectral line.

27. (b): PE of electron in second excited state in Hydrogen atom

So, second excited $n_1 = 3$

Total energy in second excited state,

$TE = \frac{-13.6}{n^2} = \frac{-13.6}{9} = -1.5\,eV$

We know, TE = PE + KE

and PE = −2 × KE

$\Rightarrow$ −1.51 = −2 KE + KE

$\Rightarrow$ −KE = −1.51 eV

$\because$ PE = −2 KE = −2 × 1.51

PE = −3.02 eV

So, option (b) is correct.

28. (a): $\frac{1}{\lambda_1} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = R\left(1 - \frac{1}{9}\right) = 8\frac{R}{9} = 0.88R$

$\frac{1}{\lambda_2} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = R\left(\frac{1}{9} - \frac{1}{16}\right) = \frac{7R}{144} = 0.0486R$

For first excited state of hydrogen atom, $n = 2$

$\therefore T = \frac{2\times6.6\times10^{-34}\times0.53\times10^{-10}}{9\times10^9\times1\times(1.6\times10^{-19})^2} = 3.03\times10^{-16}$ s

32. Here $\Delta E = 12.75$ eV

Energy of an electron in n^{th} orbit of hydrogen atom is

$E_n = \frac{-13.6}{n^2}$ eV

Energy of an electron in the excited state after absorbing a photon of 12.75 eV energy becomes

$E_n = -13.6 + 12.75 = -0.85$ eV

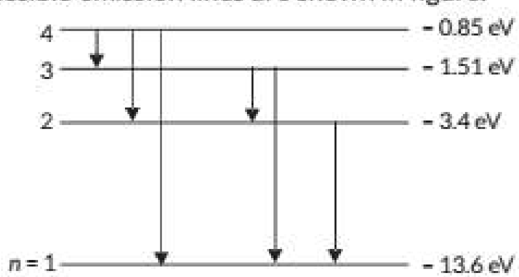
thus $n^2 = \frac{-13.6}{E_n} = -\frac{13.6}{-0.85} = 16$ or $n = 4$

Thus the electron gets excited to $n = 4$ state.

$\therefore$ Total number of wavelengths in emission spectrum

$= \frac{n(n-1)}{2} = \frac{4\times3}{2} = 6$

The possible emission lines are shown in figure.



Emitted wavelength, for the jump from initial energy

$$\frac{1}{\lambda_3}=R\left(\frac{1}{1^2}-\frac{1}{2^2}\right)=R\left(1-\frac{1}{4}\right)=3\frac{R}{4}=0.75R$$

$$\frac{1}{\lambda_4}=R\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=R\left(\frac{1}{4}-\frac{1}{9}\right)=\frac{5R}{36}=0.139R$$

Energy $\propto \frac{1}{\lambda}$, so energy is highest for I.

29. (c) : The (I) transition is of absorption. Thus maximum energy photon transition will take place in transition (III) as

$$E = -3.4 - (-13.6) = 10.2 \text{ eV}$$

30. Balmer series for a hydrogen atom lies in visible region.

$\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$, where n = 3, 4, ... ∞

For minimum wavelength, $n = \infty$

$$\frac{1}{\lambda_{min}}=\frac{R}{4} \text{ or } \lambda_{min}=\frac{4}{R}$$

For maximum wavelength, $n = 3$

$$\frac{1}{\lambda_{max}}=R\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=\frac{5R}{36} \Rightarrow \lambda_{max}=\frac{36}{5R}$$

$$\frac{\lambda_{max}}{\lambda_{min}}=\frac{36}{5\times4}=\frac{9}{5}$$

31. As $T=\frac{2\pi r}{v}$

or, $T=\frac{2\pi r\cdot nh}{2\pi kZe^2}=\frac{nhr}{kZe^2}$ $\left(\because v=\frac{2\pi kZe^2}{nh}\right)$

level E_i to final energy level E_f.

$$\lambda_{if}=\frac{hc}{E_i-E_f}=\frac{6.6\times10^{-34}\times3\times10^8}{E_i-E_f}=\frac{19.8\times10^{-26}}{E_i-E_f}\text{m}$$

Wavelength emitted for the jump from level 3 to level 2.

λ_{32} = 6547.6 Å

Wavelength emitted for the jump from level 3 to level 1.

λ_{31} = 1023.6 Å

Wavelength emitted for the jump from level 2 to level 1.

λ_{21} = 1213.2 Å

Lyman series – λ_{21} (1213Å) and λ_{31} (1024 Å)

Balmer series – λ_{32} (6548Å)

33. The energy levels of H_2 atom is given as

$$E_n=\frac{-13.6}{n^2}$$

$$\Rightarrow -1.51=\frac{-13.6}{n^2}$$

$$\Rightarrow n^2=\frac{13.6}{1.51}\approx9 \Rightarrow n=3$$

$$E_n=\frac{-13.6}{n^2} \Rightarrow -3.4=\frac{-13.6}{n^2}$$

$$n^2=\frac{13.6}{3.4} \Rightarrow n^2=4 \Rightarrow n=2$$

Thus an electron makes a transition from n = 3 energy level to n = 2 energy level.

$$\therefore \frac{hc}{\lambda_{32}}=\frac{me^4}{8\varepsilon_0^2h^2}\left(\frac{1}{n_2^2}-\frac{1}{n_3^2}\right)$$

$$\frac{hc}{\lambda_{32}}=21.76\times10^{-19}\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$$

$$\lambda_{32}=\frac{hc}{21.76\times10^{-19}\left(\frac{1}{4}-\frac{1}{9}\right)}$$

$$=\frac{6.625\times10^{-34}\times3\times10^8\times36}{21.76\times10^{-19}\times5}$$

$$=\frac{715.5\times10^{-26}}{108.85\times10^{-19}}=6.57\times10^{-7}\text{ m}$$

It belongs to Balmer series.

34. The minimum energy, required to free the electron from the ground state of the hydrogen atom, is known as ionization energy of that atom.

$$\frac{1}{\lambda_{min}}=R_H\left[\frac{1}{2^2}-\frac{1}{\infty}\right]=\frac{1.097\times10^7}{4}$$

$\therefore$ $\lambda_{min} = 3.646\times10^{-7}$ m = 3646 Å

This wavelength lies in visible region of electromagnetic spectrum.

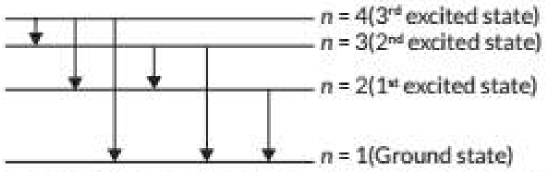
38. (i) When the electron in a H-atom jumps from higher energy level to lower energy level then spectral lines of different wavelengths are obtained for transition of electron between two different energy levels, which are found to fall in a number of spectral series.

(a) Lyman series : Emission spectral lines corresponding to the transition of electron from higher energy levels (n_2 = 2, 3, ..., ∞) to first energy level (n_1 = 1) constitute Lyman series.

$E_0 = \frac{me^4}{8\varepsilon_0^2 h^2}$ i.e., $E_0 \propto m$, so when electron in hydrogen atom is replaced by a particle of mass 200 times that of the electron, ionization energy increases by 200 times.

35. Number of spectral lines obtained due to transition of electron from $n = 4$ (3rd excited state) to $n = 1$ (ground state) is

$$N = \frac{(4)(4-1)}{2} = 6$$



The transition lines ending to $n = 1$ corresponds to lyman series, lines ending to $n = 2$ corresponds to Balmer series and the lines ending to $n = 3$ corresponds to Paschen series.

36. (a) $\lambda = 496$ nm $= 496 \times 10^{-9}$ m

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{496 \times 10^{-9}} \text{ J}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{496 \times 10^{-9} \times 1.6 \times 10^{-19}} = 2.5 \text{ eV}$$

This energy corresponds to the transition $D(n = 4$ to $n = 2)$ for which the energy change = 11.89 eV

(b) Energy of emitted photon is given by,

$$E = \frac{hc}{\lambda} \quad \therefore \lambda_{max} \propto \frac{1}{E_{min}}$$

Transition A, for which the energy emission is minimum, corresponds to the emission of radiation of maximum wavelength.

37. Wavelength (λ) of Balmer series is given by

$$\frac{1}{\lambda} = R_H\left[\frac{1}{2^2} - \frac{1}{n_i^2}\right] \text{where } n_i = 3, 4, 5, ...$$

For shortest wavelength, when transition of electrons take place from $n_i = \infty$ to $n_f = 2$ orbit, wavelength of emitted photon is shortest.

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right], \text{ where } n_2 = 2, 3, 4, ..., \infty$$

(b) Balmer series : Emission spectral lines corresponding to the transition of electron from higher energy levels ($n_2 = 3, 4, ... \infty$) to second energy level ($n_1 = 2$) constitute Balmer series.

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right], \text{ were } n_2 = 3, 4, 5, ..., \infty$$

(ii) The longest wavelength of the Lyman series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3R}{4} \text{ or } \lambda = \frac{4}{3R}$$

The shortest wavelength of the Balmer series is given by

$$\frac{1}{\lambda'} = R\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] = \frac{R}{4} \text{ or } \lambda' = \frac{4}{R}$$

Hence, $\frac{\lambda}{\lambda'} = \frac{4}{3R} \times \frac{R}{4} = \frac{1}{3}$

39. Rydberg's formula for wavelengths of the spectral lines of the hydrogen atom spectrum is given by,

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right],$$

where R = Rydberg's constant = 1.0973×10^7 m^{-1}

The emission spectrum of hydrogen, H_α line (656.3 nm) lies in Balmer series.

40. For longest wavelength of Lyman series $n_i = 2$

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$$

$$\lambda_{max} = \frac{4}{3R} = \frac{4}{3 \times 1.097 \times 10^7} = 1.215 \times 10^{-7} \text{m}$$

λ_{max} = 1215 Å

The lines of the Lyman series are found in ultraviolet region.

For longest wavelength of Balmer series $n_i = 3$

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5}{36}R$$

$$\lambda_{max} = \frac{36}{5R} = \frac{36}{5 \times 1.097 \times 10^7} = 6.563 \times 10^{-7} \text{ m} = 6563 \text{ Å}$$

Balmer series lie in the visible region of electromagnetic spectrum.

41. Here, ΔE = 12.5 eV

Energy of an electron in n^{th} orbit of hydrogen atom is,

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

In ground state, $n = 1$

$E_1 = -13.6$ eV

Kinetic energy of electron in ($n = 2$) state is

$K_2 = -E_2 = +3.4$ eV

(b) Radius in the first excited state

$r_1 = (2)^2 (0.53)$ Å

$r_1 = 2.12$ Å

43. According to Bohr's atomic model, the circumference of the electron orbit is always an integral multiple of de-Broglie wavelength.

Energy of an electron in the excited state after absorbing a photon of 12.5 eV energy will be

$E_n = -13.6 + 12.5 = -1.1$ eV

$\therefore \quad n^2 = \frac{-13.6}{E_n} = \frac{-13.6}{-1.1} = 12.36 \Rightarrow n = 3.5$

Here, state of electron cannot be fraction.

So, $n = 3$ (2^{nd} exited state).

The wavelength λ of the first member of Lyman series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3}{4}R$$

$$\Rightarrow \lambda = \frac{4}{3R} = \frac{4}{3 \times 1.097 \times 10^7}$$

$$\Rightarrow \lambda = 1.215 \times 10^{-7} \text{ m}$$

$$\Rightarrow \lambda = 121 \times 10^{-9} \text{ m} \Rightarrow \lambda = 121 \text{ nm}$$

The wavelength λ' of the first member of the Balmer series is given by

$$\frac{1}{\lambda'} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36}R$$

$$\Rightarrow \lambda' = \frac{36}{5R} = \frac{36}{5 \times (1.097 \times 10^7)}$$

$$= 6.56 \times 10^{-7} \text{ m} = 656 \times 10^{-9} \text{ m} = 656 \text{ nm}$$

42. (i) $\because E_n = \frac{-13.6}{n^2}$ eV

Energy of the photon emitted during a transition of the electron from the first excited state to its ground state is,

$\Delta E = E_2 - E_1$

$$= \frac{-13.6}{2^2} - \left(\frac{-13.6}{1^2}\right) = \frac{-13.6}{4} + \frac{13.6}{1} = -3.40 + 13.6$$

$= 10.2$ eV

This transition lies in the region of Lyman series.

Concept Applied

- Energy of n^{th} orbit of an atom is $E_n = -13.6\frac{Z^2}{n^2}$ eV

(ii) (a) The energy levels of H-atom are given by

$$E_n = -\frac{Rhc}{n^2} = -\frac{13.6}{n^2} \text{ eV}$$

For first excited state $n = 2$

$$E_2 = -\frac{13.6}{(2)^2} \text{ eV} = -3.4 \text{ eV}$$

44. From the de Broglie hypothesis

$2\pi r_n = n\lambda_n$

$$\Rightarrow \lambda_n = \frac{2\pi r_n}{n} = 2\pi n (0.529 \text{ Å}) \qquad (\because r_n = n^2 \times 0.529 \text{ Å})$$

$$\therefore \frac{\lambda_3}{\lambda_4} = \frac{2 \times 3.14 \times 3 \times 0.529 \text{ Å}}{2 \times 3.14 \times 4 \times 0.529 \text{ Å}}$$

$$\frac{\lambda_3}{\lambda_4} = \frac{3}{4}$$

45. Kinetic energy of the electron in the second state of hydrogen atom

$$E_K = \frac{13.6 \text{ eV}}{n^2} = \frac{13.6 \text{ eV}}{4} = 3.4 \times 1.6 \times 10^{-19} \text{ J}$$

de Broglie wavelength $\lambda = \frac{h}{\sqrt{2mE_K}}$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}}} = 0.67 \text{ nm}$$

46. According to Bohr's postulates,

$$mvr = \frac{nh}{2\pi} \qquad \text{... (i)}$$

(where mvr = angular momentum of an electron and n is an integer).

Thus, the centripetal force, $\frac{mv^2}{r}$ (experienced by the electron) is due to the electrostatic attraction, $\frac{kZe^2}{r^2}$

Where, Z = Atomic number

Therefore, $\frac{mv^2}{r} = \frac{kZe^2}{r^2}$

Substituting the value of v^2 from (i), we obtain:

$$\frac{m}{r}\frac{n^2h^2}{4\pi^2 m^2 r^2} = \frac{kZe^2}{r^2} \quad \therefore \quad r = \frac{n^2h^2}{4\pi^2 mkZe^2}$$

The relation for the n^{th} radius of Bohr orbit in terms of Bohr's quantization condition of orbital angular momentum $= \frac{n^2h^2}{4\pi^2 mkZe^2}$.

47. de-Broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$, where K is the kinetic energy.

Now, energy of electron,

$$K = \frac{13.6z^2}{n^2} = \frac{13.6}{3^2} = 1.51 \text{ eV} = 2.41 \times 10^{-19} \text{ J}$$

$\therefore \quad \lambda = \frac{h}{\sqrt{2mK}} = \frac{6.63\times10^{-34}}{\sqrt{2\times9\times10^{-31}\times2.41\times10^{-19}}}$

$= 1\times10^{-9}$ m = 1 nm

48. Bohr's quantization condition : The electron revolves around the nucleus only in those orbits for which the angular momentum is an integral multiple of $h/2\pi$.

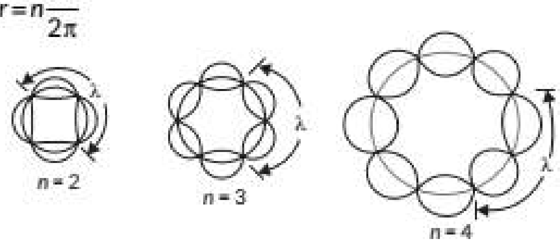
i.e., $L = mvr = n\frac{h}{2\pi}$; $n = 1, 2, 3, ...$

de-Broglie hypothesis may be used to derive Bohr's formula by considering the electron to be a wave spread over the entire orbit, rather than as a particle which at any instant is located at a point in its orbit. The stable orbits in an atom are those which are standing waves. Formation of standing waves require that the circumference of the orbit is equal in length to an integral multiple of the wavelength. Thus, if r is the radius of the orbit

$$2\pi r = n\lambda = \frac{nh}{p} \qquad \left(\because \lambda = \frac{h}{p}\right)$$

which gives the angular momentum quantization.

$$L = pr = n\frac{h}{2\pi}$$



49. (i) Bohr's quantization condition : The electron revolves around the nucleus only in those orbits for which the angular momentum is an integral multiple of $h/2\pi$.

i.e., $L = mvr = n\frac{h}{2\pi}$; $n = 1, 2, 3, ...$

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which gives the angular momentum quantization.

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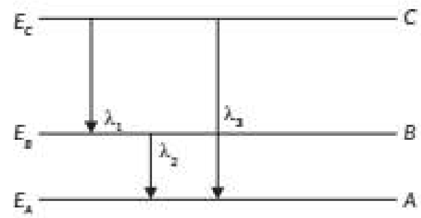
(ii)



Clearly, from energy level diagram,

$E_C - E_A = (E_C - E_B) + (E_B - E_A)$

(On the basis of energy of emitted photon).

$$\frac{hc}{\lambda_3} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$\Rightarrow \frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \Rightarrow \lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}$$

which is the required relation between the three given wavelengths.

50. Kinetic energy in the first excited state of hydrogen atom

$E_K = 3.4$ eV $= 3.4\times1.6\times10^{-19}$ J

De Broglie wavelength, $\lambda = \frac{h}{\sqrt{2m E_K}}$

$$= \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times3.4\times1.6\times10^{-19}}} = 0.67 \text{ nm}$$

CBSE Sample Questions

1. (d): Given, $r = 5.3\times10^{-11}$ m

Let r_1 be the radius at $n = 3$,

$r_1 = n^2 r = (3)^2\times5.3\times10^{-11}$ m $= 4.77\times10^{-10}$ m **(1)**

2. No, because according to Bohr's model, $E_n = -\frac{13.6}{n^2}$ **(1)**

and electrons have different energies for different values of n.

So, their angular momenta will be different, as

$L = mvr = \frac{nh}{2\pi}$ **(1)**

3. The angular momentum of an electron in Bohr orbit is given as

$$L = \frac{nh}{2\pi}$$

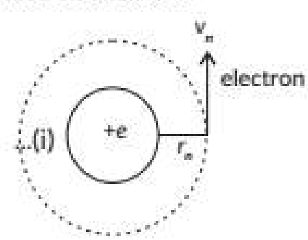
It is an integral multiple of $h/2\pi$.

Therefore, the angular momentum in second orbit $(n = 2) = 2h/2\pi = h/\pi$ **(1)**

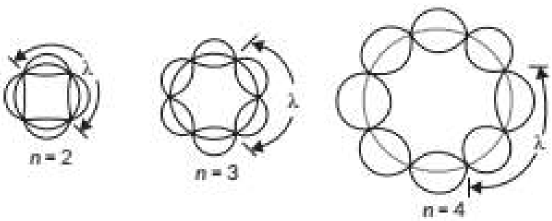
4. Centripetal force = electrostatic attraction

$$\frac{mv_n^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2}$$

$$mv_n^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n}$$



As $mv_n r_n = n\cdot\frac{h}{\pi}$



Put the value of v_n in eqn. (i)

$$m\cdot\frac{n^2h^2}{4\pi^2m^2r_n^2}=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} \Rightarrow r_n=\frac{\varepsilon_0 n^2h^2}{\pi me^2} \quad (2)$$

5. Energy of electron in $n = 2$ is -3.4 eV

K.E. = –T.E.

$$E_n=\frac{x}{n^2} \Rightarrow -3.4\,\text{eV}=\frac{x}{2^2}$$

$\Rightarrow \quad x = -13.6$ eV

Energy in ground state, $x = -13.6$ eV, K.E. = 13.6 eV (1)

P.E. = 2T.E. = $-2 \times 13.6 = -27.2$ eV (2)

6. For a transition from $n = 3$ to $n = 1$ state, the energy of the emitted photon,

$$h\upsilon = E_3 - E_1 = 13.6\left[\frac{1}{1^2}-\frac{1}{3^2}\right]\text{eV} = 12.1\text{ eV.}$$

From Einstein's photoelectric equation,

$h\upsilon = K_{max} + W_0$

$\therefore \quad W_0 = h\upsilon - K_{max} = 12.1 - 9 = 3.1$ eV

Threshold wavelength,

$$\lambda_{th}=\frac{hc}{W_0}=\frac{6.62\times10^{-34}\times3\times10^8}{3.1\times1.6\times10^{-19}}=4\times10^{-7}\text{ m} \quad (3)$$

7. (b) : As per question, $12.1 = 13.6 - \frac{13.6}{n^2}$

$\Rightarrow \quad \frac{13.6}{n^2} = 1.5$

$\Rightarrow \quad n^2 = 9$

$\Rightarrow \quad n = 3$ (1)

As $mv_nr_n = n\cdot\frac{h}{2\pi}$

$$v_n=\frac{nh}{2\pi mr_n}$$

8. Given, short wavelength limit of Lyman series,

$$\frac{1}{\lambda_L}=R\left(\frac{1}{1^2}-\frac{1}{\infty}\right) \Rightarrow \frac{1}{913.4\text{ Å}}=R\left(\frac{1}{1^2}-\frac{1}{\infty}\right)$$

$$\lambda_L=\frac{1}{R}=913.4\text{ Å}$$

For the short wavelength limit of Balmer series,

$n_1 = 2, n_2 = \infty$

$$\frac{1}{\lambda_B}=R\left(\frac{1}{2^2}-\frac{1}{\infty}\right) \Rightarrow \lambda_B=\frac{4}{R}=4\times913.4\text{ Å}=3653.6\text{ Å} \quad (2)$$

9. From Bohr's theory, the frequency υ of the radiation emitted when an electron de-excites from level n_2 to level n_1 is given as

$$\upsilon=\frac{E_2-E_1}{h}$$

$$\upsilon=\frac{me^4}{8\varepsilon_0^2h^3}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

Given $n_1 = n - 1, n_2 = n$,

$$\upsilon=\frac{me^4}{8\varepsilon_0^2h^3}\frac{2n-1}{(n-1)^2n^2} \quad (2)$$

For large n, $2n - 1 = 2n$, $n - 1 = n$

Thus, $\upsilon=\frac{me^4}{4\varepsilon_0^2h^3n^3}$

$$\upsilon=\frac{v}{2\pi r}=\frac{me^4}{4\varepsilon_0^2h^3n^3} \quad (1)$$

which is same as orbital frequency of electron in n^{th} orbit.